THE NATURE OF THE SPECTRAL GAP FOR LEAKY WAVES ON A PERIODIC STRIP GRATING STRUCTURE

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ABSTRACT

The nature of the spectral gap at forward endfire for a periodic leaky-wave structure consisting of an infinite array of metallic strips on a lossless grounded dielectric layer is studied and compared with that for a simple grounded dielectric layer. One basic difference is that the nature of the spectral gap depends on whether or not a second space harmonic (n=-2) begins radiating before the main radiating harmonic (n=-1) is scanned to forward endfire. The spectral gap resembles that for a *lossy* dielectric layer when a second space harmonic is radiating, and resembles that for a *lossless* dielectric layer otherwise. In addition, an *interesting new solution*, which has no counterpart in the dielectric layer case, is found to exist when the n=-2 harmonic is nonradiating in the scan range of the n=-1 harmonic.

INTRODUCTION

The spectral gap region is a transition region in frequency where a guided mode changes from a physically meaningful leaky mode to a bound surface-wave mode. This transition region typically occurs as the beam radiated by the leaky mode is scanned to endfire. An understanding of the spectral gap region is important in order to obtain a physical description of how the various radiating modes on a structure evolve. The specific guiding structure that will be analyzed is the periodic strip-grating structure shown in Fig. 1. It consists of a periodic array of infinitesimally thin perfectly conducting strips on a lossless grounded substrate. The structure is designed so that the fundamental (n=0) space harmonic is a slow wave. Radiation occurs from the n=-1 space harmonic. However, there may be two space harmonics, n=-1 and n=-2, that

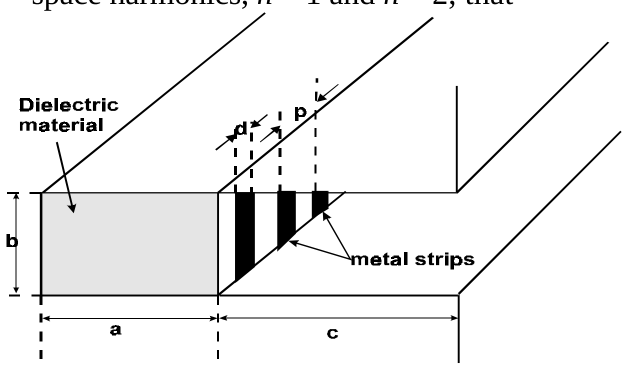

Fig. 1 Leaky-wave antenna based on a metal strip loaded dielectric inset waveguide.

radiate simultaneously depending on the strip spacing and permittivity of the substrate. In order to have single-beam operation over the entire scan range, the permittivity must be chosen large enough so that

$$\varepsilon_r > 9 + \left(\frac{p}{a}\right)^2 . \qquad (1)$$

The analysis used here to obtain the propagation constant of the structure is based on a transverse resonance procedure using a rigorous multimode equivalent network model of the grating [1]. The periodic grating excites

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an infinite set of space harmonics. A different branch choice for the transverse wavenumber k_{xn} is possible for each of the infinite number of space harmonics. The most convenient way to study the different solutions obtained by the different choice of branches is to introduce a steepest-descent variable ζ_n for each of the space harmonics. Although all of the solutions obtained from the different branch choices are valid mathematically, most of the solutions are completely nonphysical. For the space harmonics that are well into the slow-wave region $(|\beta_n| >> k_o)$, the solution will only have physical meaning provided that the *proper* branch choice is taken. Therefore, to keep the investigation tractable, the steepest descent representation has been used for only *two* of the space harmonics, -1 and -2, since two is the maximum number of space harmonics that are likely to be in, or close to, the fast-wave region.

NUMERICAL RESULTS

Using the analysis technique mentioned of [1], the propagation constants for different guided-wave solutions that were found to exist have been obtained. For all of the results shown here, the substrate thickness is a = 0.14 cm, the grating period is p = 0.338 cm, and the strip width to period ratio (d/p) is 0.2. Figure 2 shows results for the case $\varepsilon_r = 9.0$ as the frequency is scanned from 46 to 58 GHz. In this case, Eq. (1) is not satisfied and the -2 harmonic enters the fast-wave region before the -1 mode is scanned to forward endfire. Four different solutions were found from the numerical search, labeled as **A**, **B**, **C**, and **D**. The solutions for the n=-1 harmonic (ζ_{-1}) are shown on the right side of the plot, while the corresponding solutions for n=-2 (ζ_{-2}) harmonic are shown on the left side. The extreme steepest-descent paths, which denote the boundaries between the fast and slow-wave regions, are also shown with dashed lines.

One feature of the solutions is that they come in conjugate pairs. Solution (**A**) in Fig. 2 is the solution that, at lower frequencies, corresponds to the main radiating mode of the structure. This solution exhibits the characteristics of the leaky mode solution for the *lossy* dielectric layer, even though the dielectric in Fig. 1 is *lossless*. This is because radiation from the -2 harmonic introduces a loss mechanism, in much the same way as having a loss tangent does. As the frequency increases the -2 space harmonic emerges from the slow-wave region near the $-\pi/2$ line and moves toward the real axis. At 57.0 GHz, the -1 space harmonic crosses the ESDP path and enters the spectral gap region. At this frequency, the -2 mode is well within the fast-wave region. A noticeable bump in the solution occurs before the -1 harmonic reaches the ESDP, at approximately 53.6 GHz. The frequency of this bump corresponds to mode coupling between the forward and backward space harmonics. Solution (**B**), which is the conjugate of solution (**A**), is a nonphysical solution since both the -1 and -2 harmonics stay within the nonphysical growing regions of the steepest-descent plane, where the waves increase in the longitudinal direction of power flow.

Solution (**C**) in Fig. 2 corresponds to the nonphysical second solution in the lossy dielectric layer case. This solution starts in the nonphysical growing regions, for both the -1 and -2 harmonics, at lower frequencies. As the frequency increases the harmonics cross the real axis at the same frequency of 56.6 GHz. The -1 harmonic enters the slow wave region at 56.94 GHz, and the solution then corresponds to a physical surface-wave solution. Solution (**D**), the conjugate of solution (**C**), has the -2 harmonic in a non-physical region since it corresponds to an improper backward wave (physical backward waves should be proper). Hence solution (**D**) is not regarded as being physically significant at any frequency.

In summary, solutions (**A**) and (**C**) are the only ones that have portions of their curves lie in physically meaningful regions of the steepest-descent plane, and these two solutions exchange physical meaning as the frequency is increased. At lower frequencies solution (**A**) is a physical leaky-mode solution, while at higher frequencies solution (**C**) is a physical surface-wave solution.

As the permittivity of the substrate is increased to $\varepsilon_r = 20$, the frequency characteristics of the four solutions change significantly, as shown in Fig. 3. In this case, Eq. (1) is satisfied and the -2 mode of solution (**A**) stays within the slow-wave region until the -1 harmonic reaches forward endfire at 34.15 GHz. The behavior of the n=-1 harmonic in the spectral gap resembles that for the *lossless* dielectric layer. A *splitting point* now occurs at 34.15 GHz, where solutions (**A**) and (**B**) merge, as do solutions (**C**) and (**D**). One interesting point is that solution (**A**) loses physical meaning at around 34.15 GHz, while solution (**C**) picks up physical meaning only after 37.4 GHz (since this is the frequency at which both harmonics are in a physical region). Hence there is a noticeable frequency region of about 3 GHz, over which none of the four solutions have physical meaning.

These surprising observations lead to a closer investigation of the solutions near the spectral gap region for high permittivity substrates. It was found, after careful examination, that a *new solution* exists in the frequency range where none of the previous four solutions have physical meaning. This new solution is shown in Fig. 4 as **E** (dashed curve), which shows a magnified plot of the spectral-gap region for the $\varepsilon_r = 20$ case of Fig. 3 (the solutions (**A**) and (**D**) are omitted in this figure to enhance the clarity). The new solution exists between 34.15 and 37.4 GHz. Below 34.15 GHz, the new solution merges with solution (**B**), and above 37.4 GHz it merges with solution (**C**). The -1 harmonic of the new solution lies on top of that of solution (**C**) between 34.15 and 37.4 GHz. However, the -2 harmonic of the new solution is a physical wave (backward and proper) while the -2 harmonic of solution (**C**) is not physical in this frequency range (it is backward and improper). Hence, only the new solution is physical in this frequency range. The frequency range over which the new solution exists becomes vanishingly small as the permittivity of the substrate approaches the critical value approximately predicted by Eq. (1), when the leakage rate is small.

CONCLUSIONS

The present study has revealed a number of new and interesting features. The nature of the spectral gap and the number of solutions that participate in the spectral gap depends on whether or not a second harmonic begins to radiate before the main harmonic reaches forward endfire. If it does, there are four solutions that are observed and the spectral gap resembles that of a *lossy* dielectric layer. Physical meaning shifts from one of the solutions (a physical leaky-mode solution) to another solution (one that becomes a physical surface-wave solution). If the second harmonic does not radiate before the main harmonic reaches forward endfire, the spectral gap resembles that of a *lossless* dielectric layer and a *new solution* is found. It is this solution that becomes physically meaningful after the leaky-mode solution first reaches forward endfire and enters the spectral gap region.

REFERENCES

[1] M. Guglielmi and A. A. Oliner, "Multimode network description of a planar periodic metal-strip grating at a dielectric interface - Part I: Rigorous network formulations", *IEEE Trans. Microwave Theory Tech.*, Vol. MTT-37, pp. 534-541, March 1989.

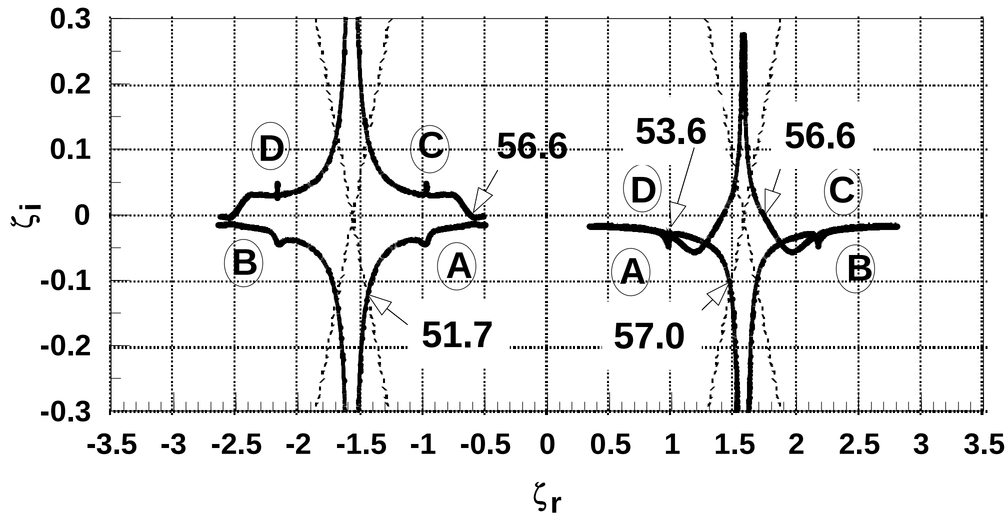

Fig. 2 A steepest descent plot showing the four different solutions for the case of ε_r=9 as the frequency is scanned from 46 to 58 GHz.

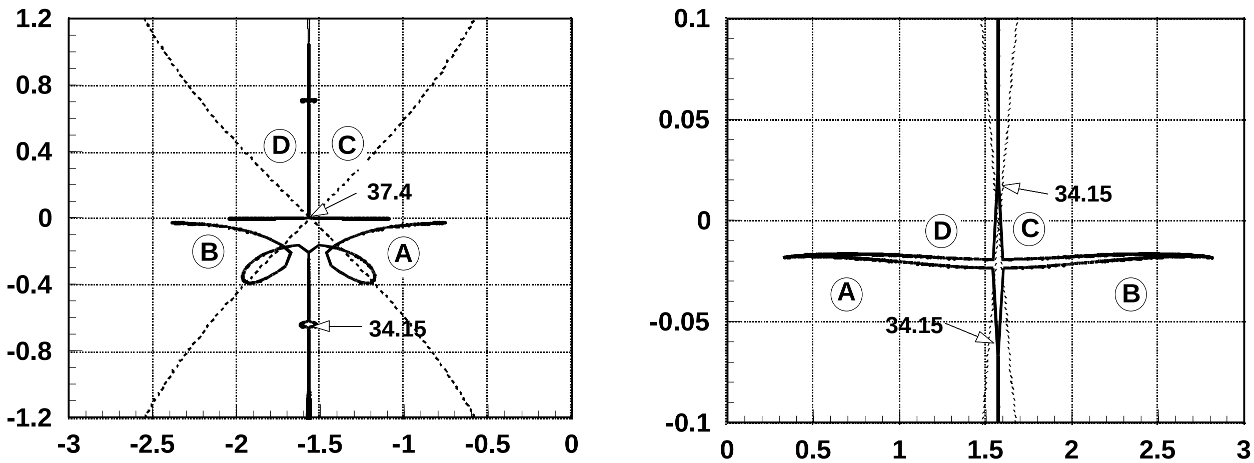

Fig. 3 A steepest-descent plot showing the four solutions for the case of ε_r=20 .

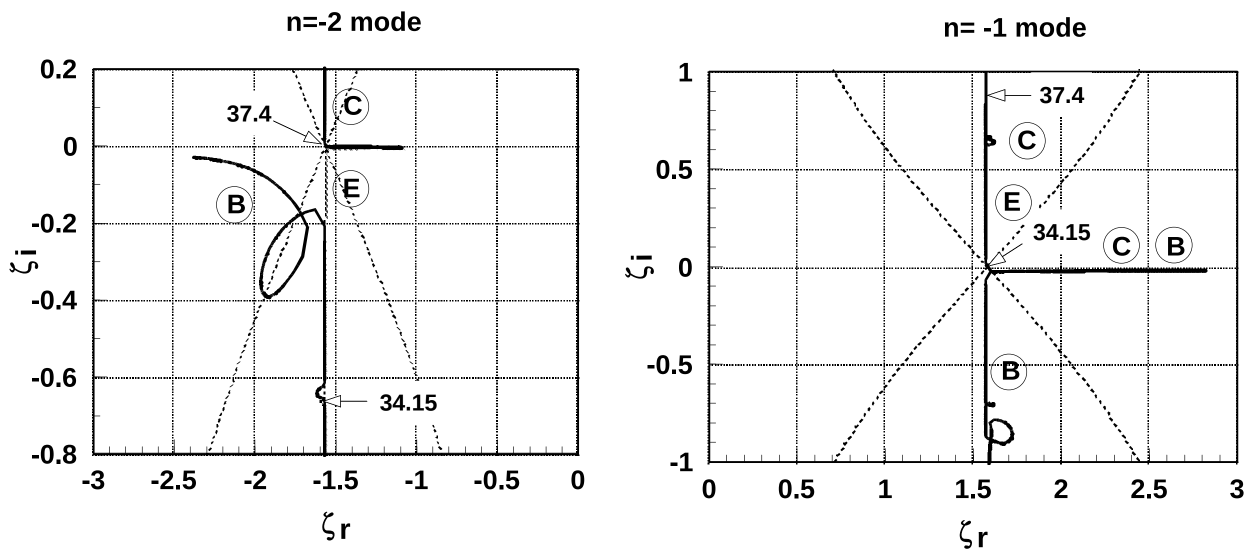

Fig. 4 A plot of the spectral gap region for the case of ε_r=20 showing the new solution (E).

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